N Dimensional Orthogonal QPSK Signaling with Discrete Prolate Spheroidal Sequences

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Abstract

A relatively new modulation method, previously referred to as Q^2PSK, is described in further detail. An analysis for system bandwidth and bit error performance is given, along with results when the set of Discrete Prolate Spheroidal Sequences are used as the optimal pulse shaping functions. Particular attention is given to performance under channel bandlimiting, nonlinear transformations, and fading conditions.

Key words: QPSK, Discrete Prolate Spheroidal Sequences, system bandwidth, bit error performance

I. Introduction

A recent extension of QPSK called Q^2PSK [1] doubles the number of signal dimensions from two for QPSK to four. Saha and Birdsall also mentioned that if n orthogonal pulse shapes were available then an extension of this system is possible which operates in a $2n$ dimensional signal space. Contributions of this paper include further analysis for this n dimensional QPSK signaling method, also referred to as Q^nPSK, and the use of DPSSs as the pulse shaping functions in a Q^nPSK system.

For discrete-time sequences, the set of Discrete Prolate Spheroidal Sequences (DPSSs) are known to comprise the most bandwidth-efficient set of index-limited sequences possible [2]. The DPSSs form an orthogonal set, so the set of DPSSs appears to be an excellent choice for the pulse-shaping functions of a Q^nPSK system. Although DPSSs have been difficult to generate in the past, new methods have recently been presented which make their generation easier [3]. The results given for the system bandwidth and error probabilities in the following sections have been produced with the first four DPSSs from the set defined by N = 64 and W = .04, where N is the number of DPSSs in the set and W is a bandwidth shaping factor, $0 \leq W \leq 0.5$. The power spectral densities corresponding to these sequences are shown in Figure 1.

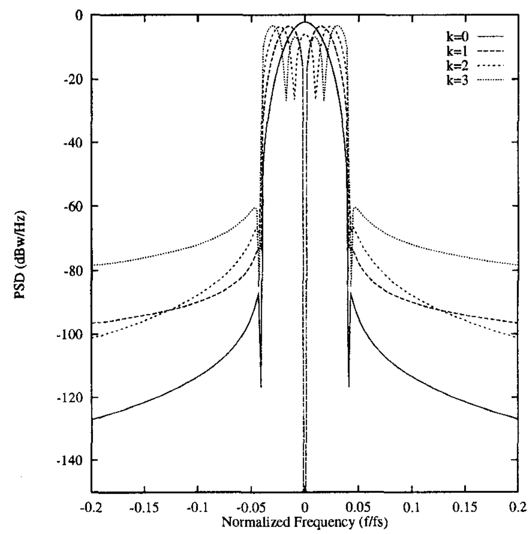

Fig. 1. Spectral densities of DPSSs $N = 64$, $W = .04$, $k = 0, 1, 2, 3$.

II. System Overview

The output of a QPSK transmitter may be represented by

$$\tilde{z}(t) = e^{j\theta^{(n)}} p(t - nT_s), \tag{1}$$

where the $e^{j\theta^{(n)}}$ represent the complex data values being transmitted and $p(t - nT_s)$ represents a pulse shaping function. For traditional QPSK the pulse shaping function $p(t)$ is rectangular in shape. Nonrectangular pulse shapes are commonly used to improve the spectral characteristics of the output signal. If we have available n orthogonal pulse shapes, a Q^nPSK system can be defined by using these n pulses in n different QPSK subsystems, with the output of each summed together for the final output. For a constant transmission rate it is important to note that the symbol period increases with n as given by $T_s = n2T$. The block diagram of a Q^nPSK system transmitter is shown in Figure 2.

The biggest advantage of Q^nPSK is its potential improved spectral efficiency over similar lower dimension systems. If the n pulses being used comprise a bandwidth-efficient set of waveforms, then the bandwidth of the Q^nPSK system may decrease

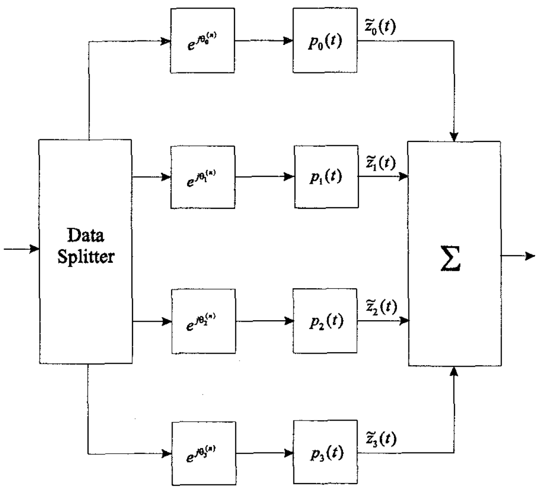

Fig. 2. Lowpass model of Q^nPSK system transmitter

appreciably as n is increased if each subsystem transmission rate is equal to$1/n$ of the original QPSK system.

III. System Power Spectra

Our objective is to find the average lowpass power spectrum of $z(t)$ indicated by $S_{\tilde{z}}(f)$, which may be found using the Wiener-Khinchin Theorem. Using methods similar to those of Titsworth and Welch [4], it can be found that the resulting average power spectrum will be

$$S_{\tilde{z}}(f) = \sum_{j=0}^{J-1} \frac{1}{T_s} P_j^*(f) P_j(f)$$
$$= \sum_{j=0}^{J-1} \frac{1}{T_s} \mid P_j(f) \mid^2 . \qquad (2)$$

The representative power spectra of several different Q^nPSK systems were found using the results above and are shown in Figure 3. All DPSS pulse shapes used were extended in length to a final length of 512 samples.

IV. Bit-Error Performance of Q^nPSK Under Non-Ideal Conditions

Although it is clear the performance of Q^nPSK under the presence of additive white Gaussian noise is the same as traditional QPSK systems, the error performance of the system under other channel degradations is not so obvious. The various other channel degradations we are interested in include bandlimiting, multipath reflections, fading, and nonlinearities. We will first derive a system model under which these degradations may be considered.

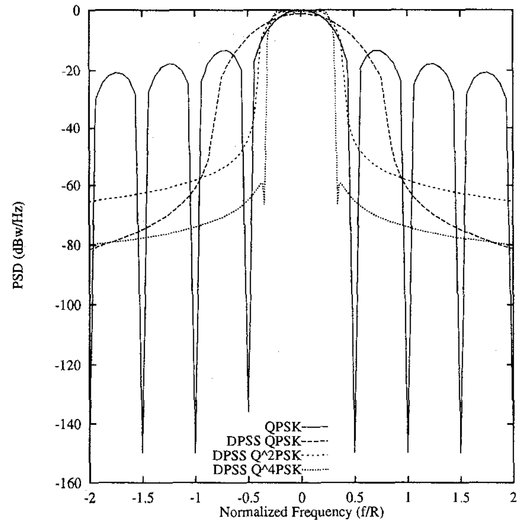

Fig. 3. Q^nPSK Power Spectral Densities
1) QPSK using traditional square pulse shapes,
2) QPSK using pulse shape DPSS k=0, N=64, W=.025,
3) Q^2PSK with DPSSs k=0,1, N=64, W=.025, and
4) Q^4PSK with DPSSs k=0,1,2,3, N=64, W=.04.

The model makes use of lowpass analysis techniques which are commonly used in studying bandpass systems [6].

The Q^nPSK transmitter in Figure 2 and corresponding matched filter receiver will be used as a guide in developing this model. Subsystems of the transmitter are referenced with j, while subsystems of the receiver are referenced with i.

Three different signals may be thought of as entering the receiver and passing through the matched filter of each subsystem. They are a result of the channel model, and are represented by:

- The direct signal, $\tilde{z}_d(t)$, with bandlimiting and nonlinearity degradations present,
- The reflected signal,$\tilde{z}_r(t)$, with the various other degradations present, and
- Channel noise, $\tilde{n}_w(t)$.

A representation for each of these signals will be found in the following sections, but first the transmitter output signal for a sequence of transmissions will be derived.

A. The Transmitted Signal

Referring to Figure 2, if the output of the j^{th} subsystem for a single transmission is denoted by $s_j(t)$, then a sequence of transmissions for that subsystem may be represented by

$$z_j(t) = \sum_{n=-\infty}^{\infty} s_j(t - nT_s). \qquad (3)$$

Using the notation of complex envelopes the composite output signal may be shown to be

$$\tilde{z}(t) = \sum_{n=-N}^{1} \sum_{j=0}^{J-1} A e^{j\theta_{n,j}} e^{-j2\pi f_c n T_s} p_j(t - nT_s). \quad (4)$$

Here $\theta_{n,j}$ denotes the phase of the complex data for the n^{th} transmission of the j^{th} subsystem.

B. The Direct Signal

If the signal of Equation 4 passes only through nonlinearities and channel bandlimiting before reaching the receiver, then this received signal is referred to as the direct signal $\tilde{z}_d(t)$. The appropriate matched filter response of the i^{th} subsystem of the receiver may be found to be

$$\tilde{H}_{r_i}(f) = \frac{A}{2} e^{-j\theta_i} P_i^*(f) e^{-j2\pi f_c (T_s)} e^{-j2\pi f (T_s)},$$

where θ_i is the phase of the receiver subsystem and a^* denotes the complex conjugate of a. If the response of the channel filtering is referred to as $\tilde{H}_c(f)$, then the i^{th} matched filter output at time T_s for the $n = 0$ transmission may be given by

$$\tilde{y}_i(T_s) = \frac{A^2}{2} e^{-j2\pi f_c (T_s)} \sum_{j=0}^{J-1} e^{j(\theta_{0,j} - \theta_i)} \cdot$$
$$\int_{-\infty}^{\infty} \tilde{H}_c(f) P_j(f) P_i^*(f) df. \quad (5)$$

For the case where the set of pulse shaping functions $p(t)$ are doubly orthogonal and no channel filtering is present we obtain the expression

$$y_i(T_s) = \begin{cases} \frac{A^2}{2\sqrt{2}} E_p, & 0 \text{ sent} \\ \frac{-A^2}{2\sqrt{2}} E_p, & 1 \text{ sent.} \end{cases} \quad (6)$$

This is the expected output for a signal which has not been degraded in any way.

C. The Reflected Signal

The reflected signal is generated by fading and multipath of the desired or direct signal. The model for the reflected signal takes on the form

$$\tilde{z}_r(t) = R(t) e^{j\phi(t)} \tilde{z}_d(t - t_d) e^{-j2\pi f_c t_d} \quad (7)$$

where $R(t)e^{j\phi(t)}$ represents a scattering model with Rayleigh amplitude and uniform phase, and t_d is average time delay. The scattering model may be represented by

$$R(t) e^{j\phi(t)} = \sum_{k=-K}^{K} b_k e^{j\lambda_k t},$$

where the b_k are statistically independent zero-mean Gaussian random variables, the λ_k are constant frequency terms, and the size of K determines the accuracy of the model [5].

The mean square value of the reflected signal processed by the i^{th} section of the receiver at the the sample time T_s can be found to be

$$E\{y_r^{2\,(i)}(T_s)\} = \sigma_r^2 = \frac{1}{4} \sum_{k=-K}^{K} \mid \tilde{y}_{r_k}^{(i)}(T_s) \mid^2 \frac{r_k}{(P_d/P_r)}, \quad (8)$$

where

$$\tilde{y}_{r_k}^{(i)}(t, t_d, \lambda_k, \ldots) = \sum_{n=-N}^{1} A \sum_{j=0}^{J-1} e^{j\theta_{n,j}} \sum_{m=-M}^{M} c_{m,j} \cdot$$
$$\tilde{H}_i(mw_o + \lambda_k) \tilde{H}_c(mw_o) e^{j(mw_o + \lambda_k)t} \cdot$$
$$e^{-j(nw_c + nmw_o)T_s} e^{-j(mw_o + w_c)t_d}, \quad (9)$$

the r_k are defined by a $2K + 1$ point Gauss quadrature rule, and P_d/P_r is the ratio of desired signal power to reflected signal power.

D. Channel Noise

The other source of noise is that which comes from the channel. Channel noise is usually modeled as having an additive white Gaussian distribution, with equivalent lowpass power spectrum

$$S_{\tilde{w}}(f) = 2N_o.$$

After passing through the i^{th} matched filter receiver the resulting power spectrum of the filtered noise $\tilde{n_i}(t)$ is

$$S_{\tilde{n}_i}(f) = 2\, N_o \, \frac{A^2}{4} \mid P_i(f) \mid^2 . \quad (10)$$

so the variance of the channel noise at the receiver is given by

$$\sigma_n^2 = N_o \, E_p \, \frac{A^2}{4}. \quad (11)$$

E. Finding P[e] using the lowpass model

For each bit permutation the conditional probability of error should be calculated for each subsystem using

$$P[e \mid D] = Q(y_i(T_s)/\sigma),$$

where $\sigma = \sqrt{\sigma_r^2 + \sigma_n^2}$. The overall system P[e] will be the average of these subsystem performance values.

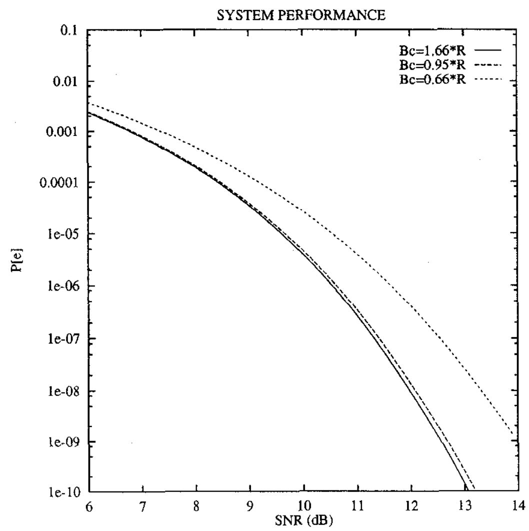

Fig. 4. Q^4PSK Performance under Bandlimiting, $B_c/R = 0.66, 0.95, 1.66$

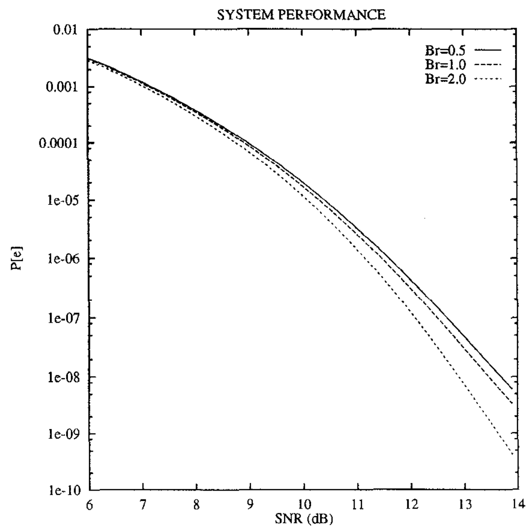

Fig. 6. Q^4PSK Performance under Fading Bandwidth B_r

F. Results Using DPSSs

The results in Figure 4 show the performance of the Q^4PSK system after bandlimiting with a four pole Butterworth filter. Figure 5 shows the system performance under nonlinear degradations with the operating level varied relative to the 1 dB compression point. Figure 6 shows the performance under fading condtions with q $P_d/P_r = 20.0$ dB, the reflected signal time delay $t_d = 1$ bit time, and fading process bandwidth B_r varied over 0.5, 1, and 2 Hz.

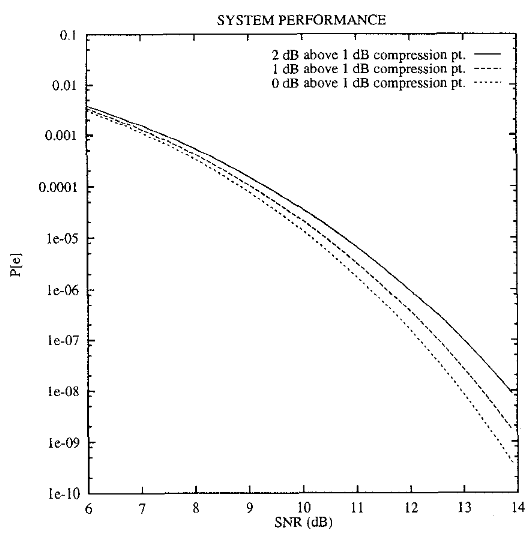

Fig. 5. Q^4PSK Performance under Nonlinear Degradations

V. Conclusions

An expanded form of QPSK called Q^nPSK is shown to have potentially greater bandwidth efficiency than conventional digital communication systems, particularly when using Discrete Prolate Spheroidal Sequences (DPSSs) as pulse shaping functions. The performance of Q^4PSK under various system degradations was studied using lowpass modeling techniques, with results showing modest improvements over current systems for most cases of channel bandlimiting.

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