HIGH RATE CODED OFDM WITH CHANNEL EQUALIZATION

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ABSTRACT

We consider the use of wideband systems for the fixed point-to-point transmission of coded data with low bit-error rate requirements. A system is defined which is based on OFDM transmission with PSK and QAM subcarrier modulation, error control coding using low-density parity-check (LDPC) codes, and channel equalization to reduce intersymbol interference from a fading channel. Tradeoffs between modulation parameters and equalization complexity are also discussed. High rate LDPC codes are considered for this system.

Keywords—LDPC, OFDM, low-density parity-check codes, fading, equalization.

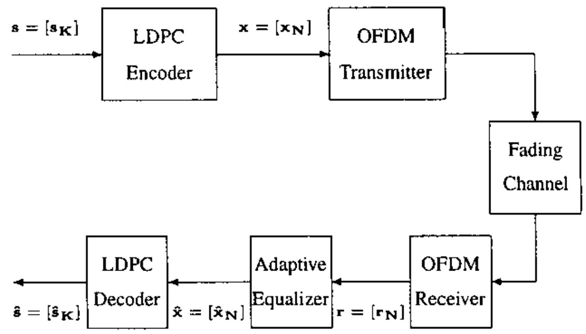

Figure 1: Channel Equalization Model

1. INTRODUCTION

Channel coding with adaptive equalization methods have been widely used in single carrier communication systems when fading and multipath propagation are present in the channel. However, difficulties could be encountered when using these techniques in systems operating at high data rates. One of the common solutions to this is Orthogonal Frequency Division Multiplexing (OFDM). When used together with an equalization technique and a powerful coding scheme significant gains over an uncoded system is obtained.

Low-density parity-check (LDPC) codes, introduced in the early 1960's by Gallager [1] and later rediscovered by MacKay and Neal [2], have been considered to be excellent error correcting codes. Their simpler decoder structure compared to that of Turbo Codes [3] have caught many researchers' attention in recent years.

This paper further extends the results analyzed in [4] and shows a performance improvement when a coded OFDM system using high rate LDPC codes also utilizes an adaptive equalizer in the decoder. The block diagram of this system is shown in Figure 1. It is assumed that there are 1024 OFDM subcarriers, each modulated by QPSK or 16-QAM.

2. LDPC CODES USING OFDM

LDPC codes are linear block codes that satisfy the equation

$$\mathbf{H}\mathbf{x} = \mathbf{0}, \tag{1}$$

where $\mathbf{H}$ is a very sparse parity-check matrix with elements from GF(q) and $\mathbf{x}$ is the codeword. With a source block length K and transmitted block length N the code rate, R, becomes $R = K/N$. This holds true if all the rows of the parity-check matrix are linearly independent and would be slightly higher if redundant rows were present. $\mathbf{H}$ is an $M \times N$ matrix with exactly weight t per column and a weight k per row, where $M = N - K$. With t and k small, $\mathbf{H}$ has a small density of ones.

The codeword $\mathbf{x}$ is obtained during the encoding process, which is defined by the linear mapping

$$\mathbf{x} = \mathbf{G}^{\mathbf{T}}\mathbf{s}, \tag{2}$$

where $\mathbf{s}$ is the source message and $\mathbf{G}$ is a $K \times N$ generator matrix obtained from the parity-check matrix $\mathbf{H}$ using Gaussian elimination.

The LDPC decoder uses the sum-product algorithm [5], also known as the belief propagation algorithm [6]. This is an iterative message-passing algorithm. The outcome of each decoding is either a success (i.e. the algorithm returns the transmitted codeword without any errors) or a failure. There are two possible types of failures: detected errors, meaning the decoding algorithm failed to find a valid codeword or undetected errors, meaning the decoding algorithm halts in a valid codeword that differs from the transmitted codeword. Refer to [7] for a detailed description of this algorithm.

OFDM has become an important part of digital communication systems due to its bandwidth efficiency and its robustness against interymbol interference (ISI). The principle behind OFDM is to split a high-rate datastream into a number of lower rate datastreams that are transmitted simultaneously over a number of subcarriers. By dividing the input datastream into N_s subcarriers, the symbol duration is made N_s times smaller. This

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reduces the relative multipath delay spread by the same factor.

ISI could be eliminated by introducing a guard time in every OFDM symbol. This is accomplished by cyclically extending the OFDM symbol into the guard time. This analysis assumes no multipath delay spread and hence does not include a guard time.

An OFDM transmitter is usually implemented using an inverse fast Fourier transform (IFFT) and the receiver using an FFT. Also, each subcarrier can be modulated by using phase shift keying (PSK) or quadrature amplitude modulation (QAM).

Since the transmitter and the receiver employ the FFT algorithm, it was suitable to pick the number of subcarriers, N_s, to be 1024. Also no channel information was estimated, other than the noise variance, but was rather assumed to be known to the receiver.

The analysis performed in [4] shows the weak performances of both random and systematically constructed LDPC codes in a 16-QAM system. Degraded performance of the 16-QAM system in a fading environment unveiled that higher modulation systems like 16-QAM would not be favorable for OFDM systems under these conditions.

To compensate for this rapid degradation, an adaptive Decision Feedback Equalizer (DFE) is placed after the OFDM receiver. This DFE uses the widely known Least Mean Square (LMS) algorithm with training sequences, thus enabling the receiver to have knowledge of the source symbols. Further details of this equalizer are described in the following section.

3. EQUALIZATION

Many forms of equalizers exist to compensate for received signal degradations due to a nonideal channel response. The primary goal of an equalizer is to remove intersymbol interference from the received signal. Traditional methods of equalization are often being replaced by newer methods that better optimize system performance by being more integrated with other key components of the receiver. For example, turbo equalization [8] is more commonly being used with systems employing iterative decoders such as turbo codes. Although LDPC codes employ an iterative decoder, this research attempts to employ the more traditional DFE equalizer in the OFDM/LDPC system.

Decision Feedback Equalization (DFE) [9] has a traditional equalizer form, with both a feedforward filter F and feedback filter G as shown in Figure 2. The length N_f FIR filter F attempts to remove the ISI associated with the k^{th} received symbol r_k, while the length N_g FIR filter G removes the ISI from previously detected symbols. The output of the equalizer is given by

$$y_k = \sum_{i=0}^{N_f} f_i r_{k-i} - \sum_{i=1}^{N_g} g_i \hat{a}_{k-i} \tag{3}$$

where f_i and f_g are the coefficients of their respective filters and $\hat{a}_k$ is the estimate of the k^{th} transmitted symbol. If the LMS form of the DFE is used, exact replicas of the transmitted symbols will be used for $\hat{a}_k$. Otherwise, the estimated symbols $\hat{a}_k$ will be determined from the detector in what is called Decision Directed (DD) [9]. Note that in the case of coded symbols, it is usually necessary to first decode the received symbols before detection. Most equalizers use the LMS form to initially train the filter coefficients, and then switch to the DD mode during the normal data transfer. Blind equalization (originally proposed in [10]) does not rely on training symbols, but instead trains the coefficients using an approach such as the Constant Modulus Algorithm (CMA) (first defined in [11]).

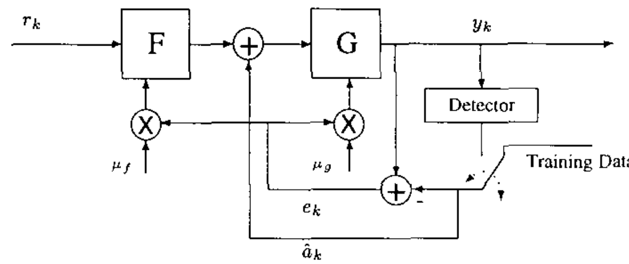

Figure 2: DFE Equalizer

An adaptive form of the DFE is shown in Figure 2. The error between the output y_k and the estimated symbol $\hat{a}_k$ is used to update the coefficients in both the feedforward and feedback filters. Scaling factors μ_f and μ_g are used to control the rate of adaptation, and they are optimally determined by the received signal-to-noise ratio. The Mean Square Error (MSE) criterion [9] is usually applied to determine the performance of the DFE. It is given by

$$\sigma_\varepsilon^2 \equiv E(a_{k-\delta} - y_k)^2, \tag{4}$$

where $a_{k-\delta} = \hat{a}_k$ for some $\delta \geq 0$. An equalizer similar to that shown in Figure 2 was inserted in our system to see if the performance in a Rician fading channel environment would be improved. Because the fading channel is considerably dynamic, the effect of the equalizer was uncertain. Several issues did arise during our implementation that affected the overall performance of the system. These issues are summarized below.

First, the use of the block-structured LDPC code with the DD equalizer adaptation did not work well in low Eb/No when fading was present. The rate of fading used, $BT_s = 0.025$ and $BT_s = 0.01$, allowed the channel response to change rapidly enough that the DFE could not keep up. Because LDPC codes must be decoded on a block by block basis, a full block of symbols must be received and decoded before they can be passed as optimum symbol estimates for the DFE. With fading present and at low Eb/No, the channel degradations were too significant for the decoder to give an adequate decoding solution for equalization. Pre-equalization of the received symbols using the equalizer configuration from the previous block was not effective because the channel was too dynamic. Only the LMS method of knowing the exact transmitted symbols worked for the presented scenarios.

One unexplored approach to this problem would be to employ the systematic structure of the LDPC codewords in a DD approach that would not require full block decoding. Optimum dispersement of the systematic bits throughout the LDPC codeword would allow the DFE to update on those symbols as they are received. Updates on the parity check symbols would not be performed. The sub-optimum equalized received block should then be more accurately decoded for a new set of detection symbols which can be used on a second iteration of the received symbols through the equalizer.

The other issue was encountered when a form of turbo equalization was attempted. In this system realization, the updated estimates generated by the LDPC decoder for the received data were fed back to the equalizer input for another iteration of processing. Using iterative equalization and decoding in this fashion did not improve the final bit-error rate over the non-iterative version.

4. RESULTS

Figures 3-6 show the performance of the system in Figure 1 for the codes in Table I. Total number of subcarriers, N_s, were 1024 and each subcarrier was either QPSK or 16-QAM modulated. In order to achieve the codeword sizes of 2048 and 4096 needed to have one LDPC codeword for each OFDM symbol the codeword sizes shown in Table I were padded with zeros. The channel was an AWGN channel with Shadowed Rician fading. The fading model was Loo's light fading model [12][13], with normalized bandwidth $BT_s = 0.025$ or $BT_s = 0.01$.

TABLE I
LDPC CODES USED IN CHANNEL SIMULATIONS

Code	M	N	Rate	Modulation
RCG	670	2010	2/3	QPSK
RCG	502	2010	3/4	QPSK
RCG	251	2010	7/8	QPSK
RCG	1340	4020	2/3	16-QAM
RCG	1005	4020	3/4	16-QAM
RCG	502	4020	7/8	16-QAM

The addition of the adaptive equalizer seemed to have made a difference in the performance of OFDM/16-QAM system as considerable gains were obtained relative to nonequalized model, especially above 10 dB. These gains could further be improved when the normalized bandwidth of the fading channel was increased.

In the case of the QPSK system some performance degradation was observed compared to the nonequalized system [4]. It is known that by decreasing the scaling factors μ_f and μ_g the convergence of the LMS algorithm is slowed somewhat, but a lower MSE and thus a performance gain is achieved. It is not certain if enough gain could have been obtained to surpass the nonequalized system since decreasing these factors in a continous manner would eventually not increase the performance.

5. CONCLUSIONS

A high rate coded OFDM system using a channel equalizer was examined in terms of its bit error rate performance. The equalizer had the DFE structure and was trained by the available source symbols to the decoder. The fading channel was constructed around two different normalized bandwidths so that the performance change between them could be observed.

The performance improvement in the 16-QAM subcarrier modulation system compared to the nonequalized system was evident. Therefore, the conclusion pointed out in [4] that the higher modulation schemes for OFDM systems under fading environment would be undesirable can be altered to include the fact that channel equalization does improve the performance of these schemes if carefully designed.

Iterative equalization/decoding of LDPC codes has been the subject of recent research [14] and is being considered for implementation in the high rate OFDM systems as future work.

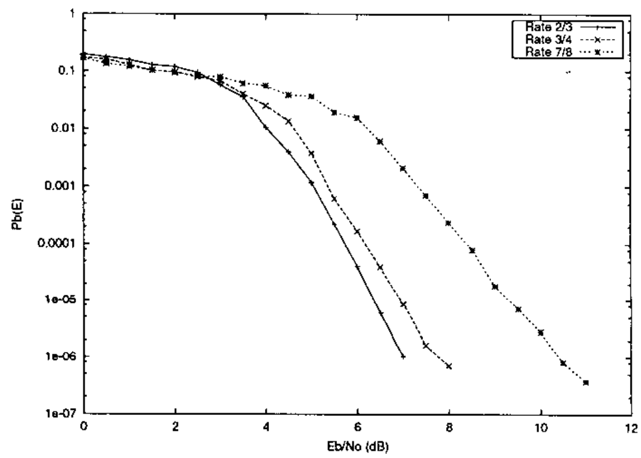

Figure 3: OFDM/QPSK vs. Code Rate, $BT_s = 0.01$

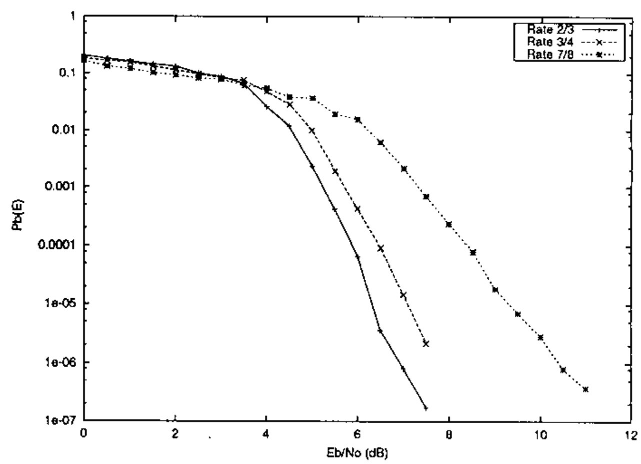

Figure 4: OFDM/QPSK vs. Code Rate, $BT_s = 0.025$

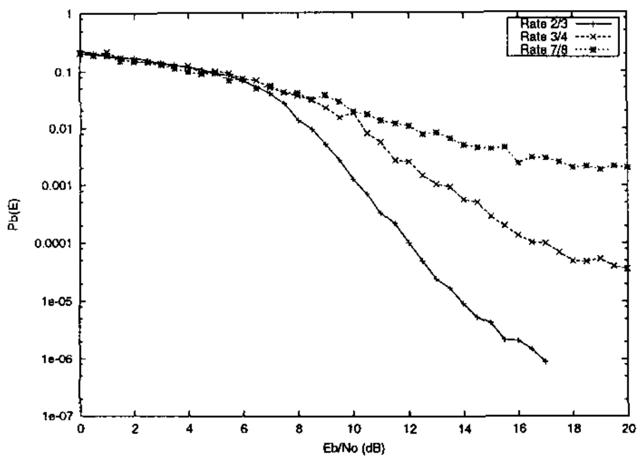

Figure 5: OFDM/16-QAM vs. Code Rate ,$BT_s = 0.01$.

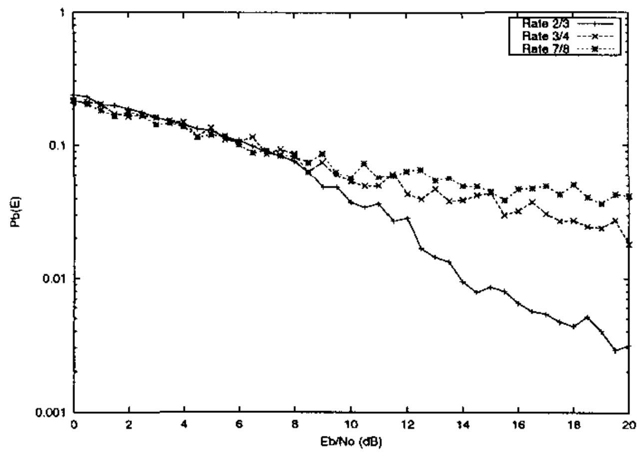

Figure 6: OFDM/16-QAM vs. Code Rate, $BT_s = 0.025$

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