This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

On the Achievability in Cooperative Amplify and Forward Wireless Relay Network

Rajet Krishnan and Balasubramaniam Natarajan
Electrical and Computer Engineering Department; Kansas State University
Telephone: (785)-532-5600; E-mail: {rajetk, bala}@ksu.edu

***Abstract*—We analyze the characteristics and performance of a wireless ad hoc network where nodes are connected via random channels and information is transported in the network in a cooperative multihop fashion using amplify and forward relay strategy. We characterize the network by studying important parameters such as: (1) SNR degradation with hop, (2) outage probability, (3) maximum permissible number of hops, and (4) maximum permissible number of simultaneous transmissions. We then devise a method for node selection and transmission of information across the network over disjoint routes between source and destination nodes by employing standard constructs from graph theory. Based on the above results, we evaluate the throughput achievable in the network and its asymptotic scaling as function of the number of nodes in the network for a given channel distribution.**

I. Introduction

Capacity and throughput analysis of wireless ad hoc networks have received considerable attention in recent times, primarily triggered by the pioneering work in [1]. The work in [1] essentially spurred research in the direction of determining theoretical bounds on the capacity of wireless networks as a function of the number of nodes for umpteen variants of the wireless network model. Protocol and physical model [1], [2], different traffic (random, symmetric, asymmetric patterns) models [3], [4], dense and extended network models [5], [6] are a few of the many instances that have been looked upon in the past.

In parallel, it has been understood that the use of multiple-input-multiple-output (MIMO) systems for wireless communication enhances spectral efficiency and link dependability significantly by virtue of spatial diversity and space-time coding. However, the direct application of MIMO systems to wireless nodes that operate in ad hoc environments has been an unattractive choice due to their inherent size and processing requirements. Instead, spatially distributed configuration of nodes has been shown to mimic a MIMO system [12]. This has been demonstrated to be possible by the formation of virtual antenna arrays through distributed transmission and signal processing, thus resulting in a form of spatial diversity, formally referred to as cooperative diversity in [7]. In [7], the performance of classical relay systems as distributed antenna arrays, using different relaying strategies and protocols, is investigated. Capacity analysis for wireless networks employing cooperative diversity (more specifically amplify-and-forward) have been analyzed exhaustively from the perspective of [1] in [8], [9].

In this paper, we consider a cooperative amplify-and-forward multihop network where the constituting nodes are connected to each other by channel strengths that are identical and independent random variables drawn from an arbitrary probability density. We base our study on the supposition that the strength of a signal received at any node in a small network is governed by random fluctuations that are not captured in the deterministic geometric models. We characterize such a random network by analyzing and evaluating important parameters like: (1) SNR degradation with hop, (2) outage probability, (3) maximum permissible number of hops and (4) source-destination node pairs that communicate with each other simultaneously. We formulate the basic operation of the network by demonstrating a scheme for choosing appropriate nodes for relaying information over disjoint routes between all source-destination node pairs. We investigate the condition of existence of such disjoint paths between all source and destination nodes in the network and their characteristics. Based on all of the above, we evaluate the achievable throughput of the network and its asymptotic scaling for channel strengths drawn from an exponential density. The premise adopted in this paper follows the model in [3], where a traditional multihop scheme using decode and forward relaying in a random network is considered. However, our work considers a cooperative amplify-and-forward multihop mode of communication as will be detailed in section II. Capacity analysis for such multi-level amplify-and-forward system has been analyzed in [11] for a fixed set of relay levels. Also in [10], the optimality of amplify-and-forward relaying strategy and the interplay between network rate, diversity and size under high SNR condition for a multi-level AF network is presented.

II. System Model

Consider a wireless ad hoc network consisting of n nodes, each equipped with single transmit-receive antenna communicating with each other in a wideband regime employing DS-CDMA. Let x_i denote the encoded message that a source node i wishes to transmit to a specific destination node j, where $x_i \sim \mathcal{CN}(0,1)\ \forall\ i,j \in \{1,\ldots,n\}, i \neq j$ and all x_i's are independent and identically distributed (i.i.d). All nodes in the network are constrained to transmit their message to any other node with a maximum power of P watts. Also, all nodes operate in a half-duplex mode where they can only either transmit or receive message at any instant of time.

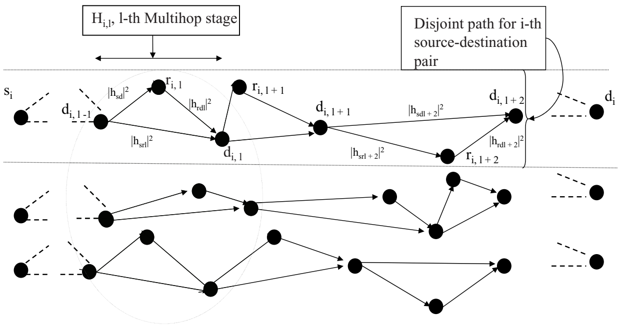

Fig. 1. System Model

Channel and Noise Model: Every pair of nodes $\{i,j\}, \forall i,j \in \{1,\ldots,n\}, i \neq j$, is connected by a symmetric frequency fading channel, denoted by $h_{i,j} = h_{j,i} = h_{ij}$, which is an i.i.d random variable. Let the channel strength $|h_{ij}|^2$ be drawn from an arbitrary probability density $f(h)$ with zero mean and variance μ. Let $n_i \sim \mathcal{CN}(0, \sigma_{wn}^2)$ denote the temporally and spatially white noise at the terminal of node i. If k source nodes, randomly chosen from $\{1,\ldots,n\}$ and denoted by set T simultaneously transmit k distinct message $x_i, i \in T$ at maximum power P, then the instantaneous signal received at the receiver of a randomly chosen destination node $j \in D = \{1,\ldots,n\} \setminus T$ is given by

$$y_j = \sum_{i=1, i \in T}^{k} \sqrt{P} h_{i,j} x_i + n_j, \quad j \in D \tag{1}$$

Here, all the signals received at j from undesirable source nodes constitute the instantaneous interference noise. Note that the interfering signals and n_j are statistically independent of the desired signal received at j.

A. Network Operation

Cooperative Multihop Communication: Consider k source nodes denoted by $s_i, i \in T$ that attempt to communicate k distinct messages $x_i, i \in T$ with k destination nodes $d_i, i \in D$. We assume that for a source-destination pair $\{s_i, d_i\}$, s_i communicates with d_i through L cooperative multihop stages denoted by $H_{i,1}, \ldots, H_{i,l}, \ldots, H_{i,L}$ as shown in figure 1. Then in the first multihop stage $H_{i,1}$, s_i communicates with an intermediate destination $d_{i,1}$ with the aid of a relay node $r_{i,1}$. In the second multihop stage, $d_{i,1}$(intermediate source) communicates with the next intermediate destination $d_{i,2}$ assisted by a relay node $r_{i,2}$ and so on, till the final destination d_i is reached after L multihop stages. Hereafter, in general we would be addressing all (intermediate and otherwise) source, relay and destination nodes as S, R, and D respectively, unless explicitly specified. We naturally impose that each cooperative multihop stage $H_{i,l}$, $l \in \{1,\ldots,L\}, L > 1$ occurs in discrete time slots $t \in \{1,\ldots,L\}$, implying that each multihop stage indexed by l is synonymous to being indexed by time slots t. Hence s_i conveys its message to the desired destination d_i in L time slots.

Protocol and Relay Strategy: In each multihop stage or time slot, S first transmits a message to R with power P. Then, both S and R simultaneously transmit to D with power P, thereby creating a virtual MISO [12]. The nodes involved in all the cooperative multihop stages for any source-destination (s_i, d_i), $\forall i \in T$ pair employ an amplify and forward relay strategy to carry message from S to D. We impose that each node in the network can assist only one source-destination pair communication in all the L multihop stages or time slots; i.e., we require k disjoint paths between all the k source-destination pairs for transporting information between them. For convenience, in general for any of the disjoint paths between source-destination pairs, we denote the channel strength between S and D, R an D, and S and R in the $l-$th stage as $|h_{sdl}|^2, |h_{rdl}|^2$ and $|h_{srl}|^2$, respectively. This is illustrated in figure 1.

Successful Communication and Throughput: Suppose S wishes to communicate with D with the assistance of R; then the communication is regarded as successful if and only if the SNR received at D is equal to or greater than a threshold SNR value ρ_0. That is, communication between any source-destination pair is successful if and only if all the constituting cooperative multihop sequences produce an SNR of at least ρ_0 at all intermediate destination nodes and the final destination node. The message is dropped in the event of unsuccessful communication in any of the multihop stages and let ϵ denote the fraction of messages dropped.

The total aggregate throughput considering k source-destination pairs is defined as [3]:

$$C = (1 - \epsilon) \frac{k}{L} \log(1 + \rho_0) \tag{2}$$

Thus, we seek to evaluate ϵ, maximum permissible value of k, L, and ρ_0 to obtain an achievability result for the throughput of the system.

III. Analysis

SNR Analysis: Based on the relaying protocol and strategy discussed in the previous section, we investigate as to how the received signal and hence the SNR at D evolves with each cooperative multihop stage or time slot. Consider a source-destination pair (s_i, d_i). For analysis, we assume that, based on some node selection rule and routing strategy, the nodes that participate in all the cooperative multihop stages or time slots $H_{i,1}, \ldots, H_{i,l}, \ldots, H_{i,L}$ have been identified. In the first cooperative multihop stage, s_i communicates its encoded message x_i with relay node $r_{i,1}$. If h_{sr1} denotes the channel connection between s_i and $r_{i,1}$, n_{r1} being the noise at $r_{i,1}$, and $h_{srj1}, j \in T \backslash \{i\}$ represents the channel between the interfering sources and $r_{i,1}$, then the signal received at the relay is -

$$y_i^{(r1)} = \sqrt{P} h_{sr1} x_i + n_{r1} + \sqrt{\rho^2} \sum_{j=1, j \neq i}^{k} \sqrt{P} h_{srj1} x_j, \tag{3}$$

The signal in (3) is normalized to unit average energy as -

$$y_{i,nor}^{(r1)} = \frac{y_i^{(r1)}}{\sqrt{P|h_{sr1}|^2 + \sigma_{wn}^2 + \rho^2 \sum_{j=1,j\neq i}^{k} P|h_{srj1}|^2}} \quad (4)$$

This is followed by a simultaneous transmission from s_i and $r_{i,1}$ to the intermediate destination $d_{i,1}$ (all with power P). If h_{sd1} and h_{rd1} denote the channel connections between s_i and $d_{i,1}$, and $r_{i,1}$ and $d_{i,1}$ respectively, n_{d1} be the noise at $d_{i,1}$, and h_{sdj1} represents the channel connection between all other sources and relays transmitting in the first stage (j indexes all other interfering sources and relays) then the signal received at $d_{i,1}$ is -

$$y_i^{(d1)} = \sqrt{P} h_{sd1} x_i + \sqrt{P} h_{rd1} y_{i,nor}^{(r1)} + n_{d1} + \sqrt{\rho^2} \sum_{j=1}^{k',j\neq i} \sqrt{P} h_{sdj1} x_j \quad (5)$$

Assuming that $d_{i,1}$ performs an equal gain combining (EGC) of the signals received from s_i and $r_{i,1}$, the amount of desired signal power $S_{i,1}$ and unwanted noise power $N_{i,1}$ at $d_{i,1}$ are as follows -

$$\begin{aligned} S_{i,1} &= P|h_{sd1}|^2 + \frac{P^2 |h_{rd1} h_{sr1}|^2}{P|h_{sr1}|^2 + \sigma_{wn}^2 + \rho^2 \sum_{j=1,j\neq i}^{k} P|h_{srj1}|^2} \\ N_{i,1} &= \frac{P|h_{rd1}|^2 (\sigma_{wn}^2 + \rho^2 \sum_{j=1,j\neq i}^{k} P|h_{srj1}|^2)}{P|h_{sr1}|^2 + \sigma_{wn}^2 + \rho^2 \sum_{j=1,j\neq i}^{k} P|h_{srj1}|^2} + \\ &\quad \sigma_{wn}^2 + \rho^2 \sum_{j=1}^{k',j\neq i} P|h_{sdj1}|^2 \end{aligned} \quad (6)$$

Instead of accounting for the instantaneous interference power at each receiver, we consider the expected value of the instantaneous interference noise power at each receiver terminal conditioned on the total number of simultaneous transmissions that occur in the stage -

$$\begin{aligned} I_1 &= \mathbb{E}\left[\rho^2 \sum_{j=1}^{k',j\neq i} P|h_{srj1}|^2 \;|k' = K\right] \\ &= \mathbb{E}\left[\rho^2 \sum_{j=1}^{k',j\neq i} P|h_{sdj1}|^2 \;|k' = K\right] \end{aligned} \quad (7)$$

We regard the number of simultaneous transmissions that occur in each stage as a random variable denoted by k' that varies between $K = (k-1)$ and $K = 2(k-1)$. By using conditionally averaged interference noise, equation (6) simplifies to -

$$\begin{aligned} S'_{i,1} &= P|h_{sd1}|^2 + \frac{P^2|h_{rd1}h_{sr1}|^2}{P|h_{sr1}|^2 + \sigma_{wn}^2 + I_1} \\ &= S_0\left(|h_{sd1}|^2 + \frac{P|h_{rd1}h_{sr1}|^2}{P|h_{sr1}|^2 + N_1}\right) \\ N'_{i,1} &= \sigma_{wn}^2 + I_1 + \frac{P|h_{rd1}|^2(\sigma_{wn}^2 + I_1)}{P|h_{sr1}|^2 + \sigma_{wn}^2 + I_1} \\ &= N_1\left(1 + \frac{P|h_{rd1}|^2}{P|h_{sr1}|^2 + N_1}\right) \end{aligned} \quad (8)$$

Where $S_0 = P$ and $N_1 = I_1 + \sigma_{wn}^2$. Using EGC, the SNR at $d_{i,1}$ after the first cooperative multihop is -

$$SNR_{i,1} = \frac{S_0\left(|h_{sd1}|^2 + \frac{P|h_{rd1}h_{sr1}|^2}{P|h_{sr1}|^2 + N_1}\right)}{N_1\left(1 + \frac{P|h_{rd1}|^2}{P|h_{sr1}|^2 + N_1}\right)} \quad (9)$$

Node $d_{i,1}$ normalizes the received signal to unit energy and then scales it by power P following which the second cooperative multihop stage $H_{i,2}$ happens as discussed in the previous section. The desired signal power and noise power at $d_{i,1}$ before transmission (after normalization and scaling to power level P) in the second time slot are as follows -

$$\begin{aligned} S_{i,1} &= S'_{i,1} \times \frac{P}{P|h_{rd1}|^2 + P|h_{sd1}|^2 + N_1} \\ N_{i,1} &= N'_{i,1} \times \frac{P}{P|h_{rd1}|^2 + P|h_{sd1}|^2 + N_1} \end{aligned} \quad (10)$$

Similarly, the signal and noise power received at $d_{i,2}$ following stage $H_{i,2}$ (after normalization and scaling) is -

$$\begin{aligned} S_{i,2} &= S_{i,1}\left(|h_{sd2}|^2 + \frac{P|h_{rd2}h_{sr2}|^2}{P|h_{sr2}|^2 + N_2}\right) \\ &\quad \times \frac{P}{P|h_{rd2}|^2 + P|h_{sd2}|^2 + N_2} \\ N'_{i,2} &= N_2\left(1 + \frac{P|h_{rd2}|^2}{P|h_{sr2}|^2 + N_2}\right) \\ &\quad + N_{i,1}\left(|h_{sd2}|^2 + \frac{P|h_{rd2}h_{sr2}|^2}{P|h_{sr2}|^2 + N_2}\right) \\ N_{i,2} &= N'_{i,2} \times \frac{P}{P|h_{rd2}|^2 + P|h_{sd2}|^2 + N_2}, \end{aligned} \quad (11)$$

where, $N_2 = I_2 + \sigma_{wn}^2$. By proceeding in the same fashion, we readily observe that after l cooperative multihop stages, the signal and noise power (after normalizing and scaling) are as follows -

$$\begin{aligned} & S_{i,l} = S_0 \prod_{x=1}^{l}\left(|h_{sdx}|^2 + \frac{P|h_{rdx}h_{srx}|^2}{P|h_{srx}|^2 + N_x}\right) \\ & \times \frac{P}{P|h_{rdx}|^2 + P|h_{sdx}|^2 + N_x}, \\ & N'_{i,l} = N_l\left(1 + \frac{P|h_{rdl}|^2}{P|h_{srl}|^2 + N_l}\right) + \sum_{x=1}^{l-1}\prod_{x=1}^{l-1} \\ & \left(|h_{sdx}|^2 + \frac{P|h_{rdx}h_{srx}|^2}{P|h_{srx}|^2 + N_x}\right)\frac{PN_{i,x}}{P|h_{rdx}|^2 + P|h_{sdx}|^2 + N_x}, \\ & N_{i,l} = N'_{i,l} \times \frac{P}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l}, \end{aligned} \quad (12)$$

where, $S_0 = P$ and $N_l = I_l + \sigma_{wn}^2$. Hence the SNR at D after l hops is given as $SNR_{i,l} = \frac{S_{i,l}}{N_{i,l}}$.

Link Formation: For any source-destination pairs (s_i, d_i), all the constituting cooperative multihop sequences must produce a minimum target SNR ρ_0 at all intermediate nodes and the final destination node. From (10), we observe that this

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

requirement is met when any S selects R and D in the $l-$th stage $\forall\, l \in \{1, \ldots, L\}$ based on the following rule:

$$\left(|h_{sdl}|^2 + \frac{P|h_{rdl}h_{srl}|^2}{P|h_{srl}|^2 + N_l}\right) \frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} \geq h_s$$
$$\bigcap \left(1 + \frac{P|h_{rdl}|^2}{P|h_{srl}|^2 + N_l}\right) \frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} \leq h_n, \quad (13)$$

where, h_s and h_n (both with units $(watt)^{-1}$) are design parameters that determine the level of connectivity and quality of connections between the nodes in the network. Note that index i has been dropped from equation (13) for convenience. Intuitively h_s gives a measure of the minimum acceptable signal power that reaches D from S in any stage; whereas h_n gives a measure of the maximum permissible noise power added in each stage. Let $p_{i,l}$ denote the probability of link formation in the l-th stage, $\forall i \in T, \forall l \in \{1, \ldots, L\}$ and hence also the probability that (13) is satisfied. Here, a link implicitly refers to a favorable S-R-D connection (a triangle in the graphical sense). Since there are k source-destination pairs communicating with each other simultaneously in any stage l, $\forall l \in \{1, \ldots, L\}$, we consider the formation of k non-overlapping favorable S-R-D connections with probability $p_{i,l}, \forall i \in T$ in each stage.

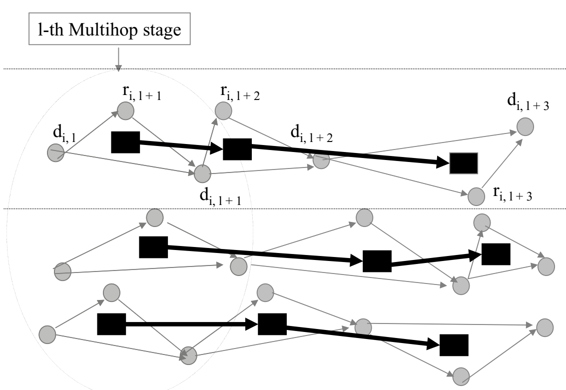

Fig. 2. Transformed Network Graph

Maximum Number of Source-Destination Pairs: We define our network of n nodes as a random graph $G(n, p'_n)$, where p'_n is the probability of link formation between any two nodes in the network. In the l-th stage of operation, $\forall l \in \{1, \ldots, L\}$, we reduce each of the S-R-D connections (triangles) participating in the $i-$th source-destination communication path to a super-node (represented by black squares) as in figure 2. Effecting the same transformation for every favorable S-R-D connections $\forall i \in T$ in all stages $\forall l \in \{1, \ldots, L\}$, we obtain a reduced graph $G'(n', p_n)$, where, $n' = k \times L$ and p_n is obtained as $p_n = p_{i,l} \times p_{i,l-1}, \forall i \in T, \forall l \in \{1, \ldots, L\}$. In order to achieve the network operation detailed in the previous section, it is required that we establish vertex-disjoint paths between all source-destination nodes in the transformed graph. The conditions for existence of such paths for k disjoint pairs of vertices (s_i, d_i) is proved in [13]. Applying the result of the paper to the transformed graph $G'(n', p_n)$, we have -

Lemma 1: Suppose that $G'(n, p_n)$ and $p_n \geq \frac{\ln n + \omega_n}{n}$, where $\omega_n \to \infty$ as $n \to \infty$, then $\exists$ a constant $\alpha > 0$ such that, with high probability, there are vertex-disjoint paths connecting (s_i, d_i) for $\forall i \in T$, such that the cardinality of set T, $|T| = k \leq \frac{\alpha n \ln n p_n}{\ln n}$. Also the length of almost all k vertex-disjoint paths are at most $\frac{\ln n}{\alpha \ln n p_n}$.

Maximum value of the minimum number of hops: We seek to evaluate an upper bound for the number of hops permissible in the system under worst case noise scenario (N) that would ensure an SNR of at least ρ_0 at D for any stage l, $\forall\, l \in \{1, \ldots, L\}$. Here, the worst case noise scenario corresponds to each node in the system experiencing maximum interference noise due to $2(k-1)$ simultaneous transmissions. First, we set -

$$\left(|h_{sdl}|^2 + \frac{P|h_{rdl}h_{srl}|^2}{P|h_{srl}|^2 + N_l}\right) \frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} = h_s$$
$$\left(1 + \frac{P|h_{rdl}|^2}{P|h_{srl}|^2 + N_l}\right) \frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} = h_n \quad (14)$$

$\forall l \in \{1, \ldots, L\}$. Then the signal power terms in (10), (11), (9) and (12) simplifies after algebraic manipulations as follows -

$$\begin{aligned} S_0 &= P \\ S_{i,1} &= S_0(Ph_s)^1 \\ S_{i,2} &= S_{i,1}Ph_s = S_0(Ph_s)^2 \\ &\vdots \\ S_{i,l} &= S_{i,0}(Ph_s)^l \end{aligned} \quad (15)$$

Similarly, the noise power terms simplify as -

$$\begin{aligned} N_{i,1} &= Nh_nP \\ N_{i,2} &= Nh_nP(1 + (Ph_s)) \\ &\vdots \\ N_{i,l} &= Nh_nP(1 + Ph_s + (Ph_s)^2 + \ldots + (Ph_s)^{l-1}) \\ &= Nh_nP\left(\frac{1 - (Ph_s)^l}{1 - (Ph_s)}\right) \end{aligned} \quad (16)$$

From the generalized signal and noise power expressions above, we can write the SNR after l cooperative multihop stages, $\forall l \in \{1, \ldots, L\}$, as -

$$\begin{aligned} SNR_{i,l} &= \frac{S_0(Ph_s)^l(1 - (Ph_s))}{Nh_nP(1 - (Ph_s)^l)} \\ &= \frac{(Ph_s)^l(1 - (Ph_s))}{Nh_n(1 - (Ph_s)^l)} \end{aligned} \quad (17)$$

Note that (17) corresponds to the minimum achievable SNR at D in any stage l, $\forall\, l \in \{1, \ldots, L\}$ for chosen values of h_s and h_n. In order to obtain an upper bound on the minimum number of hops that guarantees an SNR of at least ρ_0, we have -

$$\frac{(Ph_s)^L(1 - (Ph_s))}{Nh_n(1 - (Ph_s)^L)} \geq \rho_0 \Rightarrow L \leq \frac{\log\left(\frac{1 - (Ph_s)}{Nh_n\rho_0} + 1\right)}{\log\left(\frac{1}{Ph_s}\right)} \quad (18)$$

978-1-4244-5638-3/10/$26.00 ©2010 IEEE

The bound on L gives the maximum number of hops permissible in the system in the worst case noise scenario beyond which the SNR drops below ρ_0 and the communication is regarded unsuccessful. It can be seen that the maximum minimum number of hops directly depends upon h_s and inversely on h_n. This implies that with greater amount of noise added to the system, only smaller number of hops can ensure that the SNR at the destination is greater than ρ_0. Thus, we finally have $L \leq \min\left\{\frac{\log\left(\frac{1-(Ph_s)}{Nh_n\rho_0}+1\right)}{\log\left(\frac{1}{Ph_s}\right)}, \frac{\ln n}{\alpha \ln np_n}\right\}$ and hence the result in (24). Setting the value of L in this manner ensures that vertex-disjoint paths are established between k source-destination pairs and the threshold SNR condition is satisfied at all Ds.

Probability of Outage or Unsuccessful Communication: Communication between any source-destination pair (s_i, d_i) is regarded unsuccessful if the SNR at D in any of the L cooperative multihop sequences falls below ρ_0. If $Pr\{fail_{i,l}\} = Pr\{SNR_l < \rho_0\},\ \forall\ l \in \{1, \ldots, L\}$, then the maximum probability of erroneous communication between is given as -

$$\begin{aligned} Pr\{Fail_i\} &= \bigcup_{l=1}^{L} Pr\{fail_{i,l}\} \\ &\leq \sum_{l=1}^{L} Pr\{fail_{i,l}\} \\ &= \sum_{l=1}^{L} Pr\left\{\frac{(Ph_s)^l(1-(Ph_s))}{Nh_n(1-(Ph_s)^l)} < \rho_0\right\}, \end{aligned}$$

where, the probability of erroneous communication in each stage is considered to be independent of each other. Replacing N with N_l, we get -

$$\begin{aligned} &Pr\{Fail_i\} \geq \\ &\quad \sum_{l=1}^{L} Pr\left\{\frac{(Ph_s)^l(1-Ph_s)}{N_l h_n(1-(Ph_s)^l)} < \rho_0\right\} \\ &\quad = \sum_{l=1}^{L} Pr\left\{I_l < \frac{(Ph_s)^l(1-Ph_s)}{h_n\rho_0(1-(Ph_s)^l)} - \sigma_{wn}^2\right\} \\ &\quad = \sum_{l=1}^{L} Pr\left\{k' < \left(\frac{(Ph_s)^l(1-Ph_s)}{P\mu\rho^2 h_n\mu\rho_0(1-(Ph_s)^l)} - \frac{\sigma_{wn}^2}{P\mu\rho^2}\right)\right\} \end{aligned}$$

Hence, for successful communication, we obtain an upper bound for the maximum number of simultaneous transmissions (that determines the noise floor due to interference) in any stage $l, \forall l \in \{1, \ldots, L\}$ that depends on the orthogonality factor μ, ρ_0, P, h_s and h_n and this proves the result in (22). By setting k' to its maximum value $2(k-1)$ and $l = 1$, we readily obtain a supremum of the permissible value of ρ_0 as

$$\rho_0 \leq \frac{h_s}{h_n\left(2\mu\rho^2(k-1) + \frac{\sigma_{wn}^2}{P}\right)} \tag{19}$$

Equation (19) shows the tradeoff between connectivity and the throughput of the network. Setting high value for h_s and low value for h_n clearly enhances network throughput, as communication happens over good links resulting in relatively higher SNR at the destination nodes. However, such a configuration reduces the probability of link formation thereby resulting in a sparsely connected network.

Putting the above results together, we have the main result presented below in (21).

IV. Main Result

Theorem 1: Consider a network consisting of n nodes, where any pair of nodes $\{i, j\}$ is connected by a channel strength $|h_{i,j}|^2$ that is drawn i.i.d. from an arbitrary probability density $f(h)$. Let there be k source-destination pairs communicating simultaneously with each other over k disjoint paths via L cooperative multi-hops using amplify and forward relay strategy. For any source-destination pair (s_i, d_i), $\forall i \in T$, let any S choose R and D in the l-th stage, $\forall l \in \{1, \ldots, L\}$, based on the rule:

$$\begin{aligned} &\left(|h_{sdl}|^2 + \frac{P|h_{rdl}h_{srl}|^2}{P|h_{srl}|^2 + N_l}\right)\frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} \geq h_s \\ &\bigcap \left(1 + \frac{P|h_{rdl}|^2}{P|h_{srl}|^2 + N_l}\right)\frac{1}{P|h_{rdl}|^2 + P|h_{sdl}|^2 + N_l} \leq h_n, \end{aligned} \tag{20}$$

where, N_l denotes the sum of variance due to noisy channel and total interference noise received at D in the $l-$th stage. If $p_{i,l}$ be the probability that the condition in (20) is satisfied, then for $p_n = p_{i,l}p_{i,l-1} \geq \frac{\ln n + \omega_n}{n}, \forall i \in T, \forall l \in \{1, \ldots, L\}$, $\omega_n \to \infty$ as $n \to \infty$, $\exists \alpha > 0$, such that a throughput of

$$C = (1-\epsilon) \times \frac{k}{L} \times \log\left(1 + \frac{h_s}{h_n\left(2\mu\rho^2(k-1) + \frac{\sigma_{wn}^2}{P}\right)}\right) \tag{21}$$

is achievable where

$$\epsilon = \sum_{l=1}^{L} Pr\left\{k' < \left(\frac{(Ph_s)^l(1-Ph_s)}{P\mu\rho^2 h_n\mu\rho_0(1-(Ph_s)^l)} - \frac{\sigma_{wn}^2}{P\mu\rho^2}\right)\right\} \to 0 \tag{22}$$

$$k - 1 \leq k' \leq 2(k-1); k \leq \frac{\alpha n \ln np_n}{\ln n} \tag{23}$$

$$L \leq \min\left\{\frac{\log\left(\frac{1-(Ph_s)}{Nh_n\rho_0}+1\right)}{\log\left(\frac{1}{Ph_s}\right)}, \frac{\ln n}{\alpha \ln np_n}\right\} \tag{24}$$

and, $\rho \in \{0, 1\}$ is the orthogonality factor. It may be noted here that the maximum attainable throughput $\sup\{C\}$ for the network considered is attained upon determining optimal values of p_n, h_s, and h_n (that in turn decides k and L).

V. Simulations and Discussion

We draw $|h_{i,j}|^2$ from an exponential probability density with zero mean and variance $\mu = .5$. We set $P = 100, \sigma_{wn}^2 = 1$ and perform simulations for the evaluation of throughput of the system.We first arbitrarily fix $p_n = \frac{5 \ln n}{n}$, and evaluate $k = \frac{n \ln np_n}{\ln n}$. We then find the optimal h_s and h_n values (and correspondingly L) that maximize the throughput expression in (21). It may be noted that the throughput evaluated here

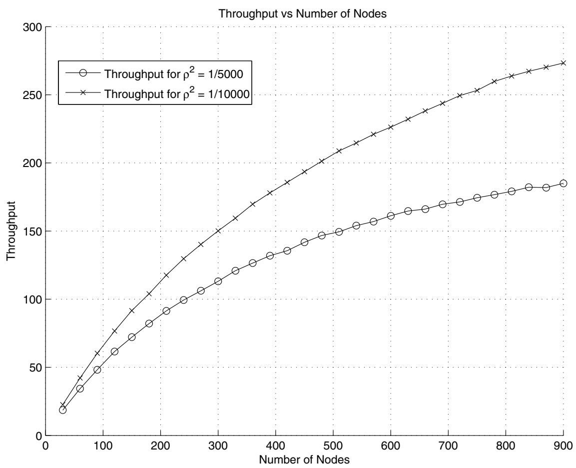

Fig. 3. Throughput of the network for different ρ^2 values.

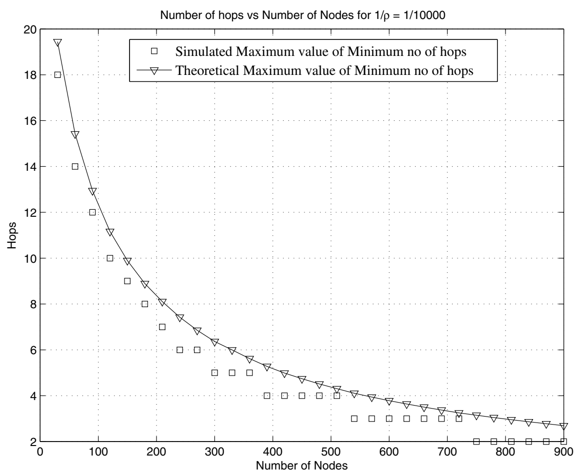

Fig. 4. Simulated and Theoretical values for maximum minimum number of hops in the network.

is not the maximum achievable value. This is because p_n has been arbitrarily picked and this may not be the optimal p_n that maximizes throughput. Monte carlo simulations are performed to statistically evaluate the throughput of the system for $n = 30$ to 900 nodes and $\rho = 1/\sqrt{5000}, 1/\sqrt{10000}$, and the results are presented in figure 3. We observe that the throughput of the system scales asymptotically as $O(\log n)$. We verify the result pertaining to maximum minimum number of hops in equation (18) through simulations for $n = 30$ to 900 nodes in figure 4. As predicted from the analytical result, we observe that the number of hops that help satisfy the SNR requirement at the destination decreases as noise increases in the system.

VI. Conclusions

In this paper, we analyze and characterize a cooperative amplify and forward multihop wireless network with random connections by studying parameters like SNR degradation, maximum permissible number of hops and source-destination pairs, outage probability, and finally the throughput achievable in the system. In order to realize the basic operation of the network, we demonstrate a scheme for node selection for transporting information over disjoint routes between any source-destination node pair in the system. Also presented are the existence condition and characterization of disjoint routes between source and destination nodes in the network. Considering the channel strengths to be drawn from exponential density, the achievable throughput appears to scale asymptotically as $O(\log n)$. From simulations, we also see that the theoretical value of the maximum value of the minimum number of hops is exact.

References

[1] P. Gupta, P. R. Kumar, "The capacity of wireless networks," IEEE Transactions on Information Theory, vol. 46, no. 2, pp. 388-404, March 2002.

[2] M. Gastpar, M. Vetterli, "On the capacity of wireless networks: the relay case," IEEE INFOCOM 2002 - Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies Proceedings, vol. 3, pp. 1577-1586, 2002.

[3] R. Gowaikar, B. Hochwald, B. Hassibi, "Communication over a wireless network with random connections," IEEE Transactions on Information Theory, vol. 52, no. 7, pp. 2857- 2871, July 2006.

[4] M. Franceschetti, O. Dousse, D. Tse, P. Tiran, "Closing the Gap in the Capacity of Wireless Networks Via Percolation Theory," IEEE Transactions on Information Theory, vol. 53, no. 3, pp. 1009-1018, March 2007.

[5] L. Xie, and P. R. Kumar, "A Network Information Theory for Wireless Communication: Scaling Laws and Optimal Operation", IEEE Transactions on Information Theory, vol. 50, no. 5, May 2004.

[6] B. Sirkeci-Mergen, A. Scaglione, "A Continuum Approach to Dense Wireless Networks with Cooperation", Proceedings of the IEEE INFOCOM, Miami, FL, March 2005.

[7] J. N. Laneman, D. N. C. Tse, G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior", IEEE Transactions on Information Theory, vol. 50, no. 12, Dec. 2004

[8] H. Blcskei, R. U. Nabar, . Oyman, A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," IEEE Transactions on Wireless Communication, vol. 5, no. 6, pp. 1433-1444, Jun. 2006.

[9] V.I. Morgenshtern, H. Bolcskei, "Crystallization in Large Wireless Networks", IEEE Transactions on Information Theory, vol. 53, no. 10, pp. 3319-3349, Oct. 2007.

[10] S. Borade, Z. Lizhong, R. Gallager, "Amplify-and-Forward in Wireless Relay Networks: Rate, Diversity, and Network Size", IEEE Transactions on Information Theory, vol. 53, no. 10, pp. 3302-3318, Oct. 2007.

[11] S. Yeh, O. Leveque, "Asymptotic capacity of multi-level amplify-and-forward relay networks," Proceedings of IEEE International Symposium on Information Theory, Nice, France, June 2007.

[12] R. U. Nabar, H. Blcskei, and F. W. Kneubhler, "Fading relay channels: Performance limits and space-time signal design," IEEE Journal on Selected Areas of Communication, vol. 22, no. 6, pp. 1099-1109, Aug. 2004.

[13] A. Z. Broder, A. M Frieze, S. Suen and E. Upfal, "An Efficient Algorithm for the Vertex-Disjoint Paths Problem in Random Graphs," Proc 7th Symp. Discrete Algorithms, Atlanta, 1996, pp. 261 268.