Residue Number System Arithmetic Inspired Hopping Pilot Pattern Design for Cellular Downlink OFDMA

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***Abstract*—In this paper, a novel hopping pilot pattern design method is proposed for downlink multi-cell OFDMA. Specifically, residue number system (RNS) arithmetic is invoked as a tool for constructing pilot patterns with equidistant sampling in both time and frequency domains, and limited mutual interference. Compared to Costas array-based pilot patterns, RNS-based method generates more unique pilot patterns, providing additional degrees of freedom for identifying cells/devices in a multi-cell multi-antenna environment.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has been widely accepted as an enabling technology for next generation wireless communication systems [1]. In OFDM, high rate data streams are broken down into a number of parallel lower-rate streams that are transmitted on orthogonal subcarriers (frequencies). OFDM also forms the foundation for a multiple access (MA) scheme termed as orthogonal frequency division multiple access (OFDMA), which is being considered for evolved 3G cellular systems E-UTRA [2]. In OFDM/OFDMA, in order to acquire channel state information (CSI) at the receiver to facilitate coherent detection, pilot assisted channel estimation techniques are generally used [3]. In pilot aided transmission scheme, a limited number of known signals referred to as pilots, are placed across time and frequency domains. Essentially, such two-dimensional pilot signals sample the channel's time-frequency response in a way that the full channel response can be reconstructed without severe aliasing. Moreover, in cellular OFDMA systems, different pilot patterns have to be allocated to different base stations while minimizing the number of collisions among the patterns. This is because if all transmitted pilot patterns are the same, they will interfere with each other in the user's equipment (UE), which in turn will degrade system throughput and the quality of channel estimation. Hence, there is a need for pilot pattern design such that, (1) the full channel response can be interpolated with a limited number of two-dimensional pilot signals. That is, in the context of OFDM, in order to fulfill Nyquist sampling theorem, there exist both a minimum subcarrier spacing and a minimum symbol spacing between pilots; (2) the number of different pilot patterns should be as large as possible with each pair of the pilot patterns having a small maximum number of collisions.

In [4], Costas array-based pilot patterns are investigated. Authors in [4] show that Costas array-based two-dimensional pilot pattern has minimal collisions with its arbitrary time shifted versions. In their later work [5], both link and system level performances of Costas array-based pilot patterns are evaluated. However, the number of unique pilot patterns that are generated via Costas array is still limited, especially when the system is not synchronized in time. Latin square-based pilot pattern design is proposed and investigated in [6] and [7]. However, wide variations in system performance (e.g., BER) are observed in Latin square-based method, which highly limits its application.

In this paper, we invoke the use of residue number system (RNS) arithmetic to determine two-dimensional pilot patterns with limited mutual interference. RNS has received broad attention in both computer computations and communications in recent years [8][9][10]. However, the use of RNS in constructing a large set of multiple two-dimensional pilot patterns with low cross-correlation has not been investigated in any of the previous works. Therefore, understanding and quantifying the role of RNS in pilot assisted channel estimation in cellular downlink OFDMA is the objective of this work.

The rest of this paper is organized as follows: RNS arithmetic is briefly introduced in section II. In section III, system model is given. Detailed construction procedures for RNS-based pilot patterns are presented in section IV. Illustrative examples are shown in section V. Finally, we conclude our paper in section VI.

II. RESIDUE NUMBER SYSTEM ARITHMETIC

Residue number system arithmetic is defined by the choice of v number of positive integers m_i ($i = 1, 2, \cdots, v$), referred to as moduli [8]. If all the moduli are pairwise relative primes to each other, any integer N_k which falls in the range of [0, M) can be uniquely and unambiguously represented by the residue sequence $(r_{k,1}, r_{k,2}, \cdots, r_{k,v})$, where $M = \prod_{i=1}^{v} m_i$. [0, M) is

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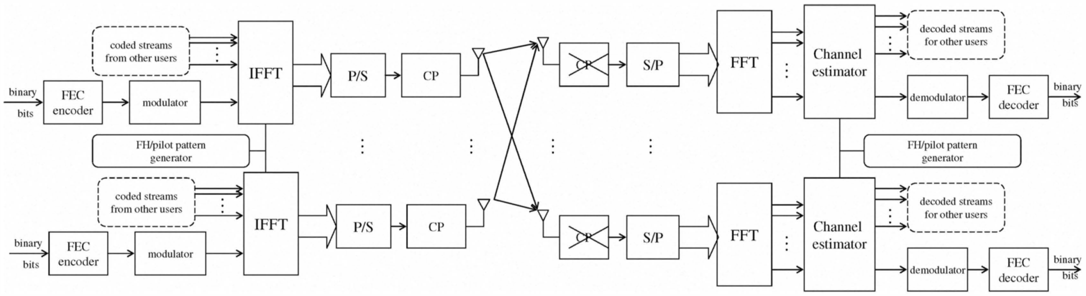

Fig. 1. Block diagram of MIMO-OFDMA system within a single cell

defined as the dynamic range of the RNS. Mathematically, above descriptions can be expressed as

$$N_k \Longleftrightarrow (r_{k,1}, r_{k,2}, \cdots, r_{k,v}), \tag{1}$$

$$r_{k,i} = N_k \mathrm{mod}\{m_i\}, i = 1, 2, \cdots, v. \tag{2}$$

III. SYSTEM MODEL

The block diagram of a multi-antenna downlink OFDMA system in a single cell is given in Fig.1. M_t transmit antennas and M_r receive antennas are employed in the system. The number of subcarriers in one OFDM block per antenna is N, including N_c subcarriers as cyclic prefix (CP), and N_p subcarriers transmitting pilot signals. We further assume that there are N_u users in the system. Each user is assigned a specific set of subcarriers out of the total available subcarriers according to his/her data rates. Let $N_{i,m}$ be the number of subcarriers allocated to user i on the $m\text{-}th$ transmit antenna. Then, user i's information symbols $\mathbf{x}^{i,m} = (x_1^{i,m}, x_2^{i,m}, \cdots, x_{N_{i,m}}^{i,m})^T$ ($(\cdot)^T$ represents the transpose operation) are transmitted on the assigned $N_{i,m}$ subcarriers. Therefore, the baseband transmitted signal for user i can be expressed as,

$$s^{i,m}(t) = \sum_{k=1}^{N_{i,m}} x_k^{i,m} e^{j2\pi \frac{k}{T_s} t}, 0 \le t < T_s, \tag{3}$$

where, $s^{i,m}(t)$ represents the time domain signal and T_s denotes one OFDM symbol duration. Moreover, zeros are transmitted on subcarriers which are not assigned to user i. For convenience, we note $C_{i,m}$ as the subcarrier set that is assigned to user i on transmit antenna m. Hence, $N \times 1$ information symbols vector of user i can be written as

$$\mathbf{x}^{i,m}(k) = \begin{cases} 0, k \notin C_{i,m} \\ x_k^{i,m}, k \in C_{i,m} \end{cases}. \tag{4}$$

The discrete form of the transmitted signal $s^{i,m}(t)$ of user i is then given as,

$$\mathbf{s}^{i,m} = \mathbf{F}\mathbf{x}^{i,m}, \tag{5}$$

where $\mathbf{F}$ is the IFFT matrix defined as

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{pmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{pmatrix} \tag{6}$$

where $W_N^{pq} = e^{j2\pi pq/N}$, $p, q = 0, 1, \cdots, N-1$.

IV. PROPOSED RNS-BASED PILOT PATTERN DESIGN

From the standpoint of sampling theory, in order to fully reconstruct the channel's delay-Doppler response, its time-frequency transform has to be sampled (at least) at Nyquist rates in both time and frequency domains. We can also interpret this in a broader sense. That is, in time domain, the sampling frequency $1/T_p$ should not be less than the channel's maximal Doppler spread $\nu_{\max}$, where T_p represents the spacing between two consecutive pilot probes in time. Similarly, in frequency domain, the sampling frequency $1/f_p$ must not be less than the channel's maximal delay spread $\tau_{\max}$, where f_p denotes the spacing between two consecutive pilot signals in frequency. In the context of OFDM, typical comb-type pilot patterns are widely used where pilot signals sample the channel's time-frequency response at Nyquist rates [11]. However, typical comb-type pilot pattern design lacks generality and this limits its application in identifying cells/devices, especially when the system is not time synchronized. In this paper, we invoke the use of RNS to design equidistant two-dimensional pilot patterns with limited mutual interference. The resulting pilot signals not only sample the channel's time-frequency response at Nyquist rates, but also generate a much larger set of unique pilot patterns relative to regular comb-type and Costas array-based pilot patterns.

Following Nyquist sampling rates, assume G as the required number of OFDM symbols between two consecutive pilot signals in time, and M as the number of OFDM subcarriers between two consecutive pilots in frequency. Therefore, if T_s is one OFDM symbol duration and f is one subcarrier spacing, we have $T_p = GT_s$ and $f_p = Nf$. Based upon those assumptions, detailed design procedures of our proposed RNS-based pilot patterns are given as follows:

1) Partition the total available subcarriers N into M_c clusters with each cluster containing M number of contiguous subcarriers (i.e., $N = MM_c$).
2) If M can be written as a product of two pairwise relative primes, e.g., $M = a \times b$, then within each cluster, we can group a sub-clusters with b subcarriers in each sub-cluster.
3) Index the subcarriers in each sub-cluster from 0 to $b-1$.
4) Index the sub-clusters in each cluster from 0 to $a-1$.
5) At the $0\text{-}th$ time slot, assign integer N_k as the initial

address (IA) of pilot signals, where $0 \leq N_k < M$.

6) If $N_k \text{mod}\{a, b\} = \{\hat{a}, \hat{b}\}$, then the $\hat{b}$-th subcarrier out of the $\hat{a}$-th sub-cluster is selected for transmitting pilot signal within one cluster.
7) Perform step 6 through all M_c clusters, M_c pilot signals are obtained with M OFDM subcarriers between each pair of the pilot signals.
8) At the t_s-th time slot, assign integer $N_k + t_s \text{mod}\{G\}$ as current address (CA) of pilot signals and repeat steps 6 - 7.
9) Repeat steps 6 - 8 until one mutually orthogonal pilot pattern is obtained.
10) If M can be expressed as products of w different combinations of two pairwise relative primes, then w different orthogonal pilot patterns can be obtained by repeating steps 2 - 9, w times.

One illustrative example of RNS-based pilot pattern design is given in Fig.2. In this example, we assume that $N = 12$, $M_c = 2$, $M = 6$ and $G = 4$. Straightforwardly, $M = 6 = 2 \times 3$. Therefore, if IA of pilot signals is 4 (at the 0-th time slot), we have $4\text{mod}\{2, 3\} = \{0, 1\}$. This indicates that the 1-st subcarrier out of the 0-th sub-cluster within each single cluster is extracted out for transmitting pilot signals. This corresponds to the actual 1-st and 7-th subcarriers in the 0-th OFDM symbol. At the 1-st time slot, the CA of pilot signals is calculated as $4 + 1\text{mod}\{4\} = 5$. Straightforwardly, $5\text{mod}\{2, 3\} = \{1, 2\}$. Hence, the 2-$nd$ subcarrier out of the 1-st sub-cluster within each single cluster is selected for pilot signals. This actually corresponds to the 5-th and 11-th subcarriers in the 1-st OFDM symbol. Fig.3 shows the corresponding pilot pattern that is generated using the RNS arithmetic described above. The IAs (i.e., 4 in this example) of pilot signals are marked at the selected positions for simplicity. In general, the entire RNS-based pilot pattern is obtained by extending the generic $M \times G$ pilot pattern in both time (see dashed bi-directional arrow) and frequency (see solid bi-directional arrow).

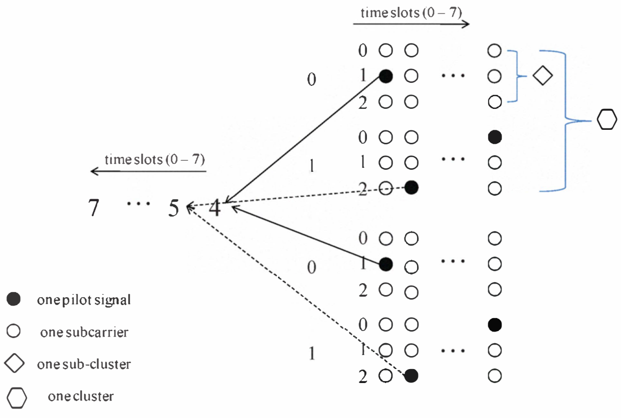

Fig. 2. Design procedures of pilot patterns using RNS arithmetic

Some important features of our proposed RNS-based pilot patterns are worth noting here:

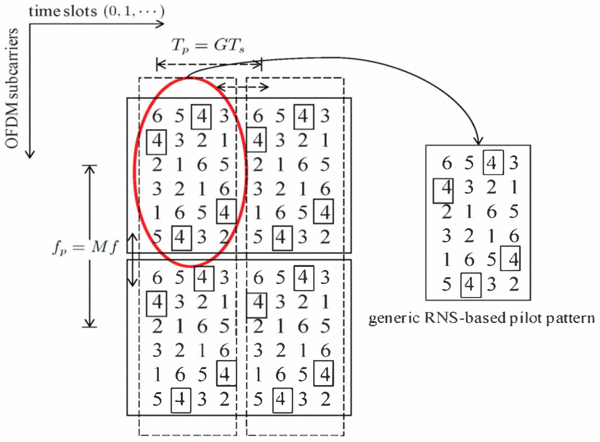

Fig. 3. Proposed RNS-based pilot pattern

Property 1: The pilot patterns designed using the RNS arithmetic as described above are orthogonal. □

Proof: Orthogonality means that pilot patterns that are obtained using different IAs, will not collide with each other. In order to prove that the pilot patterns are orthogonal, we need to show that every N_k in the range of $[0, M)$, has a unique residue set that is different from residue sets generated by other integers within the same range. We will prove this by contradiction as follows:

Assuming N_1 and N_2 are different integers which are in the same range of [0, M) with the same residue set. That is,

$$N_1 \text{mod}\{m_i\} = N_2 \text{mod}\{m_i\}, i = 1, 2, \cdots, v. \quad (7)$$

Therefore, we have

$$(N_1 - N_2)\text{mod}\{m_i\} = 0. \quad (8)$$

Thus, we can conclude from (8) that $N_1 - N_2$ is actually the least common multiple (LCM) of m_i. Furthermore, if m_i are pairwise relative primes to each other, their LCM is $M = \prod_{i=1}^{v} m_i$ and it must be that $N_1 - N_2$ is a multiple of M. However, this statement does not hold since $N_1 < M$ and $N_2 < M$. Therefore, by contradiction, N_1 and N_2 should not have the same residue set. ■

Property 2: The maximal number of collisions between any two RNS sequences under arbitrary mutual non-zero periodic time shift is one. □

Proof: In the context of OFDM, the generic pilot pattern can be represented by an integer sequence that contains indices of corresponding OFDM subcarriers across time, termed as RNS sequence. For instance, in Fig.3, the generic RNS sequence is $\{1, 5, 0, 4\}$. Therefore, in order to characterize the maximal number of collisions between each pair of the RNS sequences, we first look into the general solution that is derived based upon finite field theory, which includes RNS sequence as a special case.

An integer sequence can be defined as a set of G frequencies from a finite set of Q frequencies. If Q is prime, an one-to-one representation between the set of Q frequencies and the Galois filed of Q elements (i.e., GF(Q)) can be obtained. In other words, the integer sequence of length G ($f_i, i = 0, 1, \cdots, G-1$) can be generated by the corresponding associated polynomials of at most degree d, which is a set of elements in GF(Q) [7]. Equivalently, the integer sequence can be expressed as

$$f_i = P(i), i = 0, 1, \cdots, G-1, \tag{9}$$

$$P(x) = \sum_{j=0}^{d} n(j)x^j, \tag{10}$$

where, Q is prime; coefficient $n(d-1)$ is fixed for all associated polynomials, and all other $n(j)$ can be chosen as all possible values in GF(Q) [12].

In general, the number of collisions between two different sequences is defined as the cross-correlation of these two sequences. Assume p and r as two different integer sequences and their associated polynomials are $P(x)$ and $R(x)$, respectively. Then, the difference between these two polynomials can be calculated as

$$E(x) = P(x) - R(x) = \sum_{j=0}^{d} e(j)x^j. \tag{11}$$

From (11), we can see that $E(x)$ is at most of degree d. This indicates that sequences p and r are identical to each other at most d positions (and therefore, resulting in at most d roots of $E(x)$). From the standpoint of cross-correlation, (11) implies that the cross-correlation of two different integer sequences is no more than d, i.e., the maximal number of collisions between such two sequences is no more than d. For $d = 1$, the set of linear congruence sequences is obtained, which includes RNS sequences as a special case [8]. ■

Property 3: If the system is perfectly time synchronized, the number of RNS-based pilot patterns that have one collision with their arbitrary time shifted versions is wMG^2. For the case when the system is not synchronized in time, this number becomes wMG. □

Proof: The number of unique RNS-based pilot patterns obtained via one $M \times G$ generic pilot pattern is MG. Moreover, if M can be written as products of w different combinations of pairwise relative primes, wMG unique pilot patterns are obtained with each of them having a maximal number of collisions as one. Under perfect time synchronization, G time shifted versions can also be taken into account. Under this scenario, the number of unique RNS-based pilot patterns is wMG^2. ■

In contrast to Costas array-based pilot pattern design and typical comb-type pilot patterns, where the number of unique pilot patterns is MG^2 and MG respectively (assuming perfect time synchronization), our proposed RNS-based method significantly enhances the degree of freedom for generating unique pilot patterns, especially when w is large. If multiple antennas are employed in each of the cells, orthogonal subsets of RNS-based pilot patterns can be allocated to different transmit antennas within the same cell. This will force intra-cell interference to zero. Alternately, different RNS-based pilot patterns are allocated to adjacent cells. In this way, it is ensured that inter-cell interference can be well averaged.

TABLE I
SYSTEM PARAMETERS

Transmission BW	5MHz
Carrier frequency	2GHz
OFDM symbol duration T_s	$100\mu s$
CP duration	$10\mu s$
Subcarrier spacing f	11KHz
FFT size	256
Occupied Subcarriers	240
Number of OFDM symbols per T (G)	6
Channel impulse response	6-ray UTRA
Channel coding	1/2 convolutional code
Modulation	QPSK
Number of interference cells	5
Number of transmit antenas M_t	2
Number of receive antennas M_r	1
Pilot power boost factor ρ [5]	0dB
Time Synchronization	Perfect

V. SIMULATION RESULTS

Parameters of the simulated system is given in Table I. The cyclic prefix within one OFDM symbol duration is assumed long enough to eliminate ISI (inter-symbol interference). 6-ray channel pulse response is considered following the UTRA Vehicular Test Environment [13]. Both link and system level performances are evaluated.

A. Link level performance

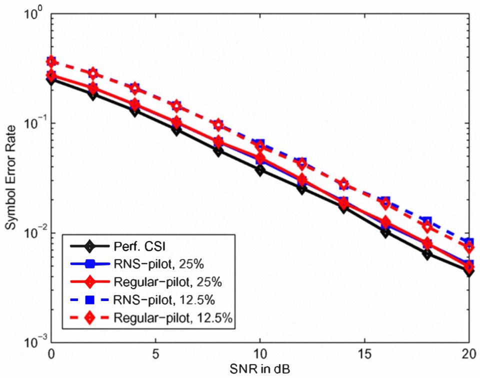

Fig. 4. SER performance of regular and RNS-based pilot pattern assisted channel estimation in a link level simulation, ρ=0dB

In Fig.4, symbol error rate (SER) performance of regular and RNS-based pilot pattern aided Least Square (LS) channel estimation is evaluated. Various pilot densities are taken into account during the simulation, where the pilot density is defined as the ratio between the number of pilot signals and the total number of subcarriers. It is observed that under the same pilot density, RNS-based pilot pattern design is

TABLE II
NUMBER OF UNIQUE PILOT PATTERNS

	No time syn.	Perf. time syn.
Regular pilot pattern	4	24
Costas-based pilot pattern	24	144
RNS-based pilot pattern	48	288

identical to regular comb-type pilot pattern in terms of SER. This is expected as the proposed RNS-based pilot pattern satisfies the sampling requirement (Nyquist rates) of typical two-dimensional pilot pattern design. Corresponding mean squared errors (MSEs) between estimated and actual channel realizations are simulated in Fig.5. From Fig.5, we conclude that RNS-based method results in the same quality of channel estimation as compared to regular comb-type pilot pattern design.

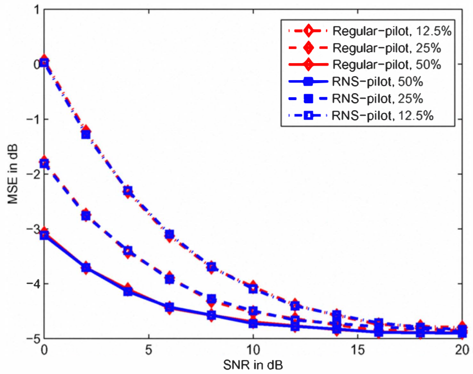

Fig. 5. MSEs of regular and RNS-based pilot pattern assisted channel estimation in a link level simulation, ρ=0dB

B. System level performance

Table II shows the number of unique pilot patterns that are generated using typical comb-type pilot pattern, Costas array-based pilot pattern and RNS-based pilot pattern. As demonstrated in section IV, RNS-based method can generate more unique pilot patterns than typical and Costas array-based pilot pattern design. Here, we assume $M = 4$ and $G = 6$. We further note that $w = 2$ as M can be written as $M = 1 \times 4 = 4 \times 1$.

In Fig.6, system throughput of RNS-based pilot pattern assisted multi-cell OFDMA is plotted under fixed SNR and SIR values. Two scenarios are considered: (1) different RNS-based pilot patterns are assigned to the cell of interest and interfering cells; (2) same RNS-based pilot patterns are allocated to the cell of interest and interfering cells. As illustrated in section IV, pilot signals usually consume higher power comparing to data signals. Therefore, if same pilot patterns are employed in adjacent cells, pilot-to-pilot collisions severely degrade the quality of channel estimation and system throughput. We can also interpret this from the perspective of frequency reuse. That is, due to the inability to generate unique pilot patterns, comb-type and Costas array-based pilot patterns suffer from higher *adjacent channel interference* (due to pilot-to-pilot collisions) as compared to RNS-based approach. This in turn, results in worse throughput performance (as square line shown in Fig.6) in contrast to RNS-based method.

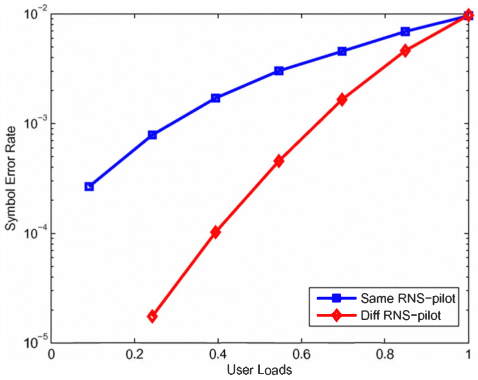

Fig. 6. System throughput of RNS-based pilot pattern assisted channel estimation in a system level simulation, SNR=20dB, SIR=15dB, ρ=0dB, 25% pilot density

VI. CONCLUSION

In this paper, a novel pilot pattern design method is proposed for downlink cellular OFDMA. Specifically, residue number system arithmetic is used as a tool for constructing uniform pilot patterns with limited interference. In contrast to typical comb-type pilot patterns and Costas array-based approach, RNS-based method results in larger set of unique pilot patterns, which is extremely helpful for identifying UEs and enhancing system throughput.

REFERENCES

[1] L. Hanzo, M. Munster, B. J. Choi, and T. Keller, “OFDM and MC-CDMA for broadband multi-user communications, WLANs and broadcasting,” in *Wiley-IEEE Press*, 2003.

[2] S. Gault, W. Hachem, and P. Ciblat, “Performance analysis of an OFDMA transmission system in a multicell enviroment,” in *IEEE Trans. on Comm.*, vol. 55, no. 4, Apr. 2007, pp. 740–751.

[3] L. Tong, B. M. Sadler, and M. Dong, “Pilot-assisted wireless transmissions: general model, design criteria, and signal processing,” in *IEEE Signal Processing Magazine*, vol. 21, no. 6, Nov. 2004, pp. 12–25.

[4] J. C. Guey, “Synchronization signal design for OFDM based on time-frequency hopping patterns,” in *IEEE ICC*, Jun. 2007, pp. 4329–4334.

[5] A. Osseiran and J. C. Guey, “Hopping pilot pattern for interference mitigation in OFDM,” in *IEEE PIMRC 2008*, Sep. 2008, pp. 1–5.

[6] “Identification of a base station, using latin-square hopping sequences, in multicarrier spread-spectrum systems,” in *European Patent Application EP 1-148-673-A3*, Oct. 2001.

[7] B. M. Popovic and Y. Li, “Frequency-hopping pilot patterns for OFDM cellular systems,” in *IEICE Trans. Fundamentals.*, vol. E89-A, no. 9, Sep. 2006, pp. 2322–2328.

[8] K. W. Watson, “Self-checking computations using residue arithmetic,” in *Proceedings of the IEEE*, vol. 54, no. 12, Dec. 1966, pp. 1920–1931.

[9] L.-L. Yang and L. Hanzo, “Redundant residue number system based error correction codes,” in *IEEE Vehicular Technology Conference*, vol. 3, 2001, pp. 1472–1476.

[10] D. Zhu and B. Natarajan, “Residue number system arithmetic assisted coded frequency-hopped OFDMA,” in *EURASIP Journal on Wireless Communications and Networking*, vol. 2009, Article ID 263695, 11 pages, doi:10.1155/2009/263695.

[11] S. Coleri, M. Ergen, A. Puri, and A. Bahai, “Channel estimation techniques based on pilot arrangement in OFDM systems,” in *IEEE Transactions on Broadcasting*, vol. 48, Sep. 2002, pp. 223–229.

[12] A. Lempel and H. Greenberger, “Families of sequences with optimum Hamming correlation properties,” in *IEEE Transactions on Information Theory*, vol. 20, 1974, pp. 90–94.

[13] *ETSI TR 101 112, UMTS 30.03, V3.1.0, Annex B, Std.*